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Minimum ATI Single Sampling Plan with Maximum Allowable Proportion of Defectives

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Abstract: This paper designs a sampling plan showing efficiency of acceptance at higher quality with lower inspection cost .The logic of MAPD as a measure is compared with Dodge Romig Plan. It brings a mathematical model for evaluating the parameters instead of Graph. The simplicity, accuracy and convenience of the model were given by examples and results.

Key words and Phrases :MAPD, LTPD, SSP, Min.ATI, OC curve.

Introduction

H.F.Dodge and H.G.Romig (1940) had developed single and double sampling plans by minimising the Average Total Inspection (ATI) for a given Lot Tolerance Percent Defective (LTPD) at a given process average. The concept of Maximum Allowable Proportion Defective (MAPD) was suggested by Mayer (1956) and further developed by Mandelson (1962), offered designs of sampling plan with better protection for both the consumer and producer. A sampling plan on MAPD and p_t , the tangent intercept was presented by Soundararajan (1975) using Poisson distribution. Ramkumar (1994) suggested unique sampling plan with MAPD and LTPD and Ramkumar and Suresh (1996) produced a sampling plan indexing MAPD and MAAOQ. R Radhakrishnan (2008) identified sampling plan on weighted Poisson distribution indexed with MAPD. Another sampling plan was marked by Ramkumar (2009) on MAPD with discriminant tangential distance. R Sampathkumar et al (2012) described mixed conditional double sampling plan on MAPD and AQL. Fuzzy numbers with a range defined on MAPD suggested by P.R. Divya (2012) is another development. Ramkumar (2013) made a critical study on importance of MAPD as a quality measure with its significances, advantages and designs. It is the distinguishable allowwable incoming quality dividing the lots into bad and good. There are many sampling designs with PAR (probability up to MAPD), after it was proposed by Ramkumar (2012) and Edokpa and Odunayo (2016) redesigned the single sampling plan on one point in the OC curve (MAPD,PAR).

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One of the conventional methods of designing sampling plan is fixing one quality on OC curve and then apply a condition of minimising cost or sample size etc. (Schilling.E.G.). Dodge and Romig table is a fundamental established sampling system in this direction fixing LTPD and minimising ATI. Dodge, making use of the concept of ATI, had sought to protect the economic aspect of modelling of a sampling plan. ATI will indirectly reduce the cost of inspection and hence the producer and consumer will be benefitted. Hald established LTPD, AQL and IQL indexed minimum cost sampling plan using the constraint on minimum cost in terms of function of ATI.

IMPORTANCE OF NEW DESIGN

Avik Ganguly concerned about the misuse and frivolous use of Dodge Romig sampling plan and expressed the need of expansion of this plan. This paper is an initiative to develop economically viable, consumer protective method of designing a sampling plan in terms of MAPD. AQL and LTPD are the qualities defined on prefixed probability and unsatisfactory in some practical situations. For example in a process under control, acceptance sampling is introduced, it is desirable to fix an incoming quality. Conventionally AQL is fixed as an incoming quality, because the process average may have 94% or 96% of probability of acceptance .So it is preferable to fix the upper bound for incoming quality at a level beyond which the quality is suddenly declined and MAPD will be suitable. Similarly it is unfair to fix final incoming quality as LTPD as it can vary the probability of acceptance from 10% or 5% to another level 8% or 3%. Since the average proportion of defectives is less than MAPD, ie $\bar{p} \le p^*$, $Pa(\bar{p}) \ge Pa(p^*)$ so that incoming quality can be prefixed at utmost allowable quality MAPD (p^* = c/n) with at least acceptable probability PAR (Pa(p^*)) . It is convenient to estimate AQL with the process average defective and then LTPD can be restricted to MAPD.

It is reasonable to form the sampling plans with parameters of control chart. If the parameter of sampling plan was fixed at MAPD, and taking this proportion of defective as the central line of control chart (process average), it will suffices to derive the conclusions on process and product average in a unique measure. Sudden fall in probability beyond p^* ensures less probability of acceptance for percent defectives (Even less than 10% at LTPD). The point of inflection implies almost same rate of acceptance for the product unto the quality p^* (Sometimes more than 95% at AQL) indicating high percent of acceptance for good items unto the quality MAPD. Thus MAPD is a balanced product quality efficient to protect both the consumer and the producer. Engineers and inspectors are highly in favour of using MAPD as it is a direct function of c & n.

Replacing LTPD by MAPD will offer better consumer protection, and the producer is benefited by a lesser-sized sampling plan. Thus MAPD-Min ATI Sampling plan is more logical, and economical. Specifically this design is admissible in the case of consumer-targeted items like electrical and electronic goods, day-to-day utility items like utensils, plastic products, kitchenware etc.

SECTION:1 BASIC PRINCIPLE

a) Methodology

Fix the incoming quality at MAPD (c/n) , process quality $\bar{p}$ and the constraint on sample size by means of min ATI at the process average. is determined. Observing the minATI from the table of ATI, corresponding c and hence n is found out. Comparing with Dodge's method, an algorithm on the min.ATI is developed and minimum c and min ATI is directly available.

b) Construction of the plan

Algorithm Method

Let p is the probability of a defective in the lot (p<0.1) follow Poisson distribution then

$$Pa(p) = \sum \frac{e^{-n^* p}(np)^r}{r!} \qquad (1)$$

ATI/at p= $\bar{p}$ = $\bar{I}$ = n+(N-n).(1-Pa ($\bar{p}$)) (2)

$$I_c = \frac{c}{p^*} + (N - \frac{c}{p^*}).\left[1 - \sum_{r=0}^{c} \frac{e^{-k.c}(kc)^r}{r!}\right] \qquad (3)$$

where k= $\bar{p}$ /p*,

which will plot the ATI for specified values of c=1,2,….40 and Min (I) give the least of

Table:1 .ATI for successive values c

n	c	Pa ($\bar{p}$)	1-Pa($\bar{p}$)	ATI=n+(N-n).(1-Pa($\bar{p}$)
20	1	0.981	0.019	115
40	2	0.991	0.009	85
60	**3**	**0.996**	**0.004**	**80**
80	4	0.998	0.002	90
100	5	0.999	0.001	105

all ATI .Then

$$c = \sum_{i}^{40} [\{I_i = \min(I)\} * i] \qquad (4)$$

will provide the corresponding minimum c, n=c/ p^* suggests the sampling plan (n,c).

Dodge's Comparison Method

Multiplying both sides by p_* and putting c=np^*, M=Np^* and Z*= $\bar{I}$ p^*

$$Z^* = M - (M - c).\sum_{r=0}^{c} \frac{e^{-n\bar{p}}(n\bar{p})^r}{r!} \qquad (5)$$

Then

$$Z^* = M - (M - c).\sum_{r=0}^{c} \frac{e^{-k.c}(k.c)^r}{r!} \qquad (6)$$

where k= $\bar{p}$ / p^*

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Substituting k and M, for various values of c the minimum value of Z* and corresponding c can be determined by trial and error method .Then min ATI , **Zmin.=Z* /p***.

SECTION:2 DISCUSSION

a) Need of minimization of ATI for fixed MAPD and process average.

Table:1 show the need of minimising Average Total Inspection since there is no meaning in increasing the sample size unnecessarily even though probability of acceptance may be increased. From the Table:1 the min. ATI is attained at c=3 with an inspection of 60 items of the sample and on average 20 items from the remaining lot.

Lot Size N=5000, $\bar{p}$ =1.05%, MAPD (p*)=5% k= $\bar{p}$ /p*=0.21

b) Finding the sampling plan

Fixing the parameters N, $\bar{p}$, p* Substitute in eqn : 3 & 4 for c=1----40, c, min.I. are directly available and given in a graph.

. From the figure:1, Min(I)=75.45=76 at c=2 and n= 2/.05=40.

By Dodges method, for k corresponding to $\bar{p}$ and p* and c=1, Plot the straight line Z_1 for the linear equation (5) for M:1---1000. Replacing c by (c+1) in equation (5), plot another straight line Z_2 for the same values of M. If there is a point of intersection between the straight lines, such value of M decides the cross over point of Z say Z*. Z will be minimum for the first line Z_1 up to M with acceptance number=1 and correspondingly find minZ=Z*/p*.Continue the process for c=2 and c=3 and respective min.Z is determined.

Figure:1 Minimum ATI and corresponding c for N=4000, $\bar{p}$ =0.0105 ,p*=0.05

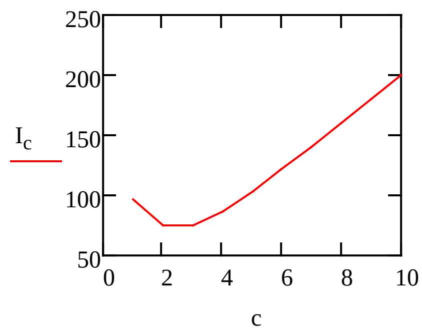
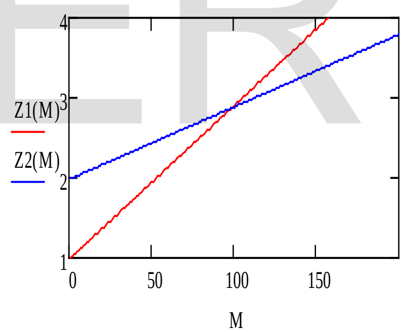


Figure:2 The Range of M for which $Z_1 < Z_2$ and $Z_1 > Z_2$.



c) Comparison of ATI

Figure: 3 Identical OC curves (N=3000, $\bar{p}$ =0.05, MAPD=0.073, LTPD=0.10)

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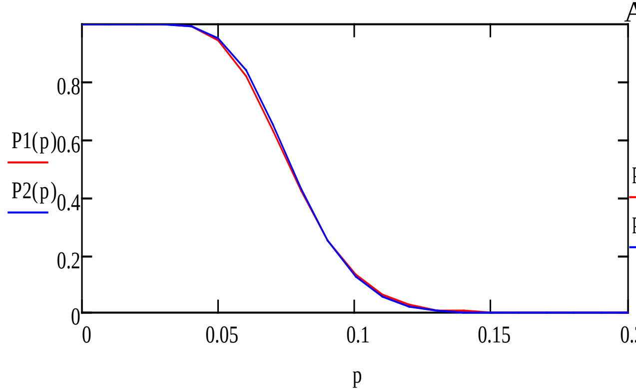
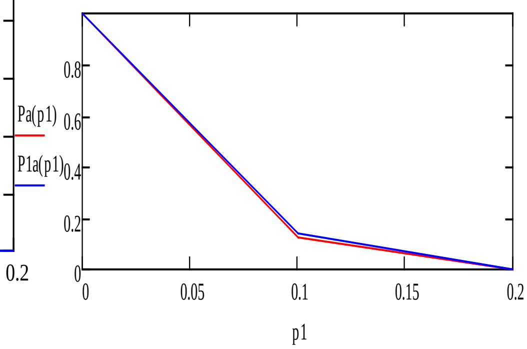
Figure: 4 OC curves with same Minimum

Lot size N	%Process average	(n)	(c)	AOQL%
7001-10000	1.01-1.50	260	8	1.9
10001-20000	1.01-1.50	285	9	2.0

ATI





1-P1(p)= (206,15) MAPD Plan, 2-P2(p)= (230,17) Dodge LTPD Plan. In Figure: 2

Table: 2 Dodge table for LTPD =5%

OC curves are identical and the minimum ATI is 371 for MAPD Plan & 357 for Dodge LTPD Plan. Thus the MAPD plan is good when the quality is maintained well as the sample size and acceptance number are less.

Fixing Minimum ATI at $\bar{p}$ =0.05 at 357 for both plans, the corresponding Dodge plan is (230,17) and MAPD Plan is (204,15) .Comparing the OC curves using figure :3, they are seemed to be identical so that the same protection is ascertained by MAPD Plan as in the LTPD Plan with less sample size and acceptance number.

d) Error in Dodge Sampling Plan

There is error in Dodge's table for estimating n,c as it is only approximated process average over a designed range.

Consider two cases below

N=7500 p_t=. 05 take $\bar{p}$ =1.02%, k=1.02/ 05 = .204, M=N.p_t=7500x.05=375.

N=18000, p_t=. 05, take $\bar{p}$ =1.42%, k=1.42/ 05 = .284,,M=N.p_t=18000.05=900.

But from Dodge table minimum Z occurs at c=8 and n=260, for N=7500, $\bar{p}$ =1.02% at 5% LTPD. But for actual substitution in equation (5) and (6) and finding the intersection, min.c = 6 for N=7500 with minimum sample size required 211. Similarly for N=18000, at same LTPD, min.c is 9 but from substitution c=11 with sample size 332. Thus there is a difference in acceptance numbers and sample sizes in the Dodge Table compared to the actual values. The actual acceptance numbers and sample size may be large or small from the tabled value.

The error has occurred because it is difficult to find sampling plan which will satisfy for all values of $\bar{p}$ over a given range. Dodges table it is approximated to middle of the range of the selected $\bar{p}$ which is only an approximation. The MAPD plan by algorithm method can directly give (n,c) for any combinations of N, $\bar{p}$,p*.

Construction of Tables

Table: 1 is developed by substituting N, $\bar{p}$,p*, in ATI formula for c=1,2,3....., Table :2 is taken from Dodge –Romig single sampling plan with respective values of $\bar{p}$,N and AOQL. Table:3a,b,c give the minimum c for the values of k=.1(.1).6 and N=100,200,500,1000,2000,5000, 10000, 20000, 50000,100000 and 0<p*<0.2 and it is obtained by substituting the said vaules in equation(3) & (4). Table:4 show optimum single sampling plan by substituting in equation (3) & (4) for given N, $\bar{p}$ p* . N=100, 200, 500, 1000, 2000, 5000, 10000,20000, 50000, 100000. and $\bar{p}$ =1% - 9% , p*=2% -10% .

Conclusions

For the identical OC curve, MAPD- Min ATI plan is more economic as the sample size is small.The acceptance number is small in MAPD- Min ATI plan so that the sampling inspection will be faster.MAPD-Min ATI plan gives mathematical solutions of c and hence n and min ATI directly. No. Need of graphical solutions to detect sampling plan. Dodge’s plan is detected only with the help of graphs. Dodge’s design required 3 graphs to obtain the minimum c. Even though ATI is less in Dodge’s method, but it will be effective only successive rejections were found. Additional sample units and rectification of remaining lots are needed to maintain low ATI. The evaluation of sample size is simple in MAPD plan. MAPD is more logical with physical interpretation and hence unique for a sampling plan, while LTPD is purely mathematical and will be altered if level of risk is changed.Dodge’s table to identify sampling plan contains errors due to intervals used.

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Table 3a: Minimum acceptance number for certain lot size with. 0.01<p*<0.20 (SSP)

N	k=. 1	k=. 2	k=. 3	k=. 4	k=. 5	k=. 6
100	p*=.2, c=1	p*=.2, c=1	p*=.2, c=1	p*=.2, c=1	p*=.2, c=1	p*=.2,c=1
200	p*=.2, c=1	p*=.2, c=1	p*=.03 ,c=1	p*=.17, c=1 p*=.2, c=2	p*=.14, c=1 p*=.2, c=2	p*=.12,c=1 p*=.16,c=2 p*=.2, c=3
500	p*=.2, c=1	p*=.17,c=1 p*=.2, c=2	p*=.09,c=1 p*=.2, c=2	p*=.06,c=1 p*=.12,c=2 p*=.2, c=3	p*=.05,c=1 p*=.08,c=2 p*=.14,c=3 p*=.2, c=4	p*=.05, c=1 p*=.06, c=2 p*=.09, c=3 p*=.15, c=4 p*=.2, c=5

Table 3b: Minimum acceptance number for certain lot size with. 0.01<p*<0.20 (SSP)

N	k=. 1	k=. 2	k=. 3	k=. 4	k=. 5		k=. 6	
1 0 0 0	p*=. 2,c=1	p*=. 1,c=1 p*=. 2,c=2	p*=.07,c=1 p*=.10,c=2 p*=.17,c=3 p*=.20,c=4	p*=.06,c=1 p*=.07,c=2 p*=.09,c=3 p*=.14,c=4 p*=.20,c=5	p*=.07,c=1 p*=.08,c=4 p*=.10,c=5	p*=.12,c=6 p*=.15,c=7 p*=.20,c=8	p*=.09,c=1 p*=.10,c=6 p*=.12,c=7 p*=.14,c=8	p*=.16,c=9 p*=.19,c=10 p*=.20,c=11
2 0 0 0	p*=.14,c=1 p*=.20,c=2	p*=05,c=1 p*=.11,c=2 p*=.20,c=3	p*=.03,c=1 p*=.05,c=2 p*=.08,c=3 p*=.15,c=4 p*=.20,c=5	p*=.03,c=1 p*=.04,c=3 p*=.05,c=4 p*=.08,c=5 p*=.11,c=6 p*=.15,c=7 p*=.20,c=8	p*=.03,c=1 p*=.04,c=4 p*=.05,c=5 p*=.07,c=6	p*=.09,c=7 p*=.11,c=8 p*=.18,c=10 p*=.20c=11	p*=.04,c=1 p*=.05,c=6 p*=.06,c=7 p*=.07,c=8 p*=.08c=9 p*=.09,c=10	p*=.10,c=11 p*=.12,c=12 p*=.14,c=13 p*=.16,c=14 p*=.19,c=15 p*=.20,c=16

5000	p*=.05,c=1 p*=.20,c=2	p*=.02,c=1 p*=.04,c=2 p*=.10,c=3 p*=.20,c=4	p*=.01,c=1 p*=.02,c=2 p*=.03,c=3 p*=.06,c=4 p*=.10,c=5 p*=.18,c=6 p*=.20,c=7	p*=.01,c=1 p*=.02,c=4 p*=.04,c=5 p*=.05,c=6 p*=.08,c=7 p*=.11,c=8 p*=.20,c=9	p*=.01,c=1 p*=.02,c=5 p*=.03,c=7 p*=.04,c=8 p*=.05,c=9 p*=.07,c=10	p*=.09,c=11 p*=.11c=12 p*=.14,c=13 p*=.18,c=14 p*=.20,c=15	p*=.01,c=1 p*=.02,c=6 p*=.03,c=9 p*=.04,c=11 p*=.05,c=12 p*=.06,c=14 p*=.07,c=15	p*=.08,c=16 p*=.09,c=17 p*=.11,c=18 p*=.12,c=19 p*=.14,c=20 p*=.16,c=21 p*=.19,c=22 p*=.20,c=23
10000	p*=.02,c=1 p*=.11,c=2 p*=.20,c=3	p*=.01,c=1 p*=.02,c=2 p*=.05,c=3 p*=.12,c=4 p*=.20,c=5	p*=.01,c=2 p*=.03,c=4 p*=.05,c=5 p*=.09,c=6 p*=.16,c=7 p*=.20,c=8	p*=.02,c=4 p*=.04,c=7 p*=.05,c=8 p*=.08,c=9 p*=.12,c=10 p*=.17,c=11 p*=.20,c=12	p*=.01,c=5 p*=.02,c=8 p*=.03,c=10 p*=.04,c=11 p*=.05,c=12 p*=.09,c=14	p*=.11,c=15 p*=.14,c=16 p*=.17,c=17 p*=.20,c=18	p*=.01,c=6 p*=.02,c=11 p*=.03,c=14 p*=.04,c=16 p*=.05,c=18 p*=.06,c=19 p*=.07,c=20	p*=.08,c=21 p*=.09,c=22 p*=.10,c=23 p*=.12,c=24 p*=.13,c=25 p*=.15,c=26 p*=.18,c=27 p*=.12,c=28

Table:4 SSP for certain parameters N, $\bar{p}$ p*(c up to 100 only)

Lot size	100	500	1000	2000	5000	10000	20000	50000	100000
$\bar{p}$ =1%,p*=2%	50,1	50,1	50,1	50,1	250,5	400,8	550,11	750,15	900,18
p*=3%	34,1	34,1	34,1	34,1	134,4	167,5	234,7	267,8	334,10
p*=4%	25,1	25,1	25,1	25,1	75,3	100,4	125,5	150,6	175,7
p*=5%	20,1	20,1	20,1	20,1	60,3	60,3	80,4	100,5	120,6
p*=6%	17,1	17,1	17,1	17,1	34,2	50,3	67,4	84,5	84,5
p*=7%	15,1	15,1	15,1	15,1	29,2	43,3	43,3	58,4	72,4
$\bar{p}$ =2%,p*=3%	33,1	33,1	33,1	33,1	367,11	600,18	834,25	1200,36	1467,44
p*=4%	25,1	25,1	25,1	100,4	200,8	275,11	350,14	450,18	525,21
p*=5%	20,1	20,1	20,1	80,4	120,6	160,8	200,10	260,13	300,15
p*=6%	17,1	17,1	17,1	50,3	84,5	117,7	134,8	167,10	184,11
p*=7%	15,1	15,1	15,1	43,3	58,4	86,6	100,7	115,8	143,10
p*=8%	13,1	13,1	13,1	38,3	50,4	63,5	75,6	88,7	101,8
$\bar{p}$ =3%,p*=4%	25,1	25,1	25,1	25,1	425,17	725,29	1075,43	1575,63	1975,79
p*=5%	20,1	20,1	20,1	120,6	240,12	360,18	460,23	600,30	720,36
p*=6%	17,1	17,1	17,1	100,6	167,10	217,13	267,16	334,20	387,23

p*=7%	15,1	15,1	15,1	72,5	115,8	143,10	172,12	265,15	258,18
p*=8%	13,1	13,1	38,3	50,4	88,7	100,8	125,10	163,13	175,14
p*=9%	12,1	12,1	34,3	45,4	67,6	78,7	100,9	123,11	134,12
$\bar{p}$ =4%,p*=5%	20,1	20,1	20,1	20,1	20,1	820,41	1260,63	1900,95	
p*=6%	17,1	17,1	17,1	17,1	300,18	417,25	550,33	734,44	884,53
p*=7%	15,1	15,1	15,1	115,8	186,13	258,18	315,22	415,29	472,33
p*=8%	13,1	13,1	50,4	88,7	138,11	175,14	213,17	263,21	313,25
p*=9%	12,1	12,1	45,4	67,6	100,9	134,12	156,14	189,17	223,20
p*=10%	10,1	10,1	40,4	50,5	80,8	100,10	120,12	150,15	170,17
$\bar{p}$ =5%,p*=6%	17,1	17,1	17,1	17,1	17,1	900,54	1417,85		
p*=7%	15,1	15,1	15,1	15,1	329,23	486,34	642,45	872,61	1043,73
p*=8%	13,1	13,1	13,1	125,10	225,18	300,24	375,30	475,38	563,45
p*=9%	12,1	12,1	56,5	100,9	156,14	200,18	245,22	312,28	367,33
$\bar{p}$ =6%,p*=7%	15,1	15,1	15,1	15,1	15,1	958,67			
p*=8%	13,1	13,1	13,1	13,1	363,28	538,43			
p*=9%	12,1	12,1	12,1	134,12	245,22	334,30	423,38	545,49	645,58
p*=10%	10,1	10,1	60,6	110,11	180,18	230.23	280,28	360,36	410,41
$\bar{p}$ =7%,p*=8%	13,1	13,1	13,1	13,1	13,1	1013,81			
p*=9%	12,1	12,1	12,1	12,1	389,35	589,53	800,72	1100,99	
p*=10%	10,1	10,1	10,1	150,15	270,27	370,37	470,47	610,61	720,72
$\bar{p}$ =8%,p*=9%	12,1	12,1	12,1	12,1	12,1	1044,94			
p*=10%	10,1	10,1	10,1	10,1	410,41	630,63	870,87		
$\bar{p}$ =9%,p*=10%	10,1	10,1	10,1	10,1	10,1				

Appendix

Result:1

If the process is in control or if the product is in control then $\bar{p} \leq p^*$

Suppose that process is in control

Let p be the proportion of defective in the lot and d the no. of defectives in the sample. Then d follows binomial distribution B (np, √npq).

i.e x/n is the proportion of defective following B (p, √pq/n)

$\bar{p}$ is the MLE of p following normal distribution, and then the central line of p chart is at $\bar{p}$. But $\bar{p}$ is calculated as ∑xi/n where each xi is either 0 or 1 as non-defectives and defectives occur with probability 1-p and p respectively,where ∑xi is the total no. of defectives 'd' in the sample.

The product will be accepted by product control using single sampling plan if d≤ c where c is the acceptance number.Thus $\sum xi/n = d/n \leq c/n = p^*$ i.e $\bar{p} \leq p^*$.

Now suppose that the product is in control, then $d \leq c \Rightarrow d/n \leq c/n = p^*$.

But d/n is the proportion of defective in the sample, which is the ratio of sum of defective units observed in the sample to the total units examined.($\sum xi/n = \bar{p}$) . By definition it is the process average in the process control. Hence the proof.

That is, average no. of defectives in the process is always less than or equal to MAPD if the lot is acceptable. Thus a statistical testing of hypothesis can be adopted on the process control and product control data to identify the condition of acceptance.

Example

A process control operation is maintained with the process average of .015 defective per unit based on a sample of 50 units and suggested sampling plan for the product is (40,1). Test whether the sampling plan is suitable for the process items.

Take the hypothesis that the sampling plan suits the data ,H_0: $p \leq p^*$ against H_1: $p > p^*$

Test statistic Z= $(\bar{p} - p^*)/\sqrt{\bar{p}\,\bar{q}/n}$ follows, standard normal distribution , Z= -.582 <1.645.accept the hypothesis, then at 5% level of significance the product control sampling plan is accepted as conforming to the process control.

Result 2

UCL of process control is bounded by MAPD if the process is under control

$$\text{UCL} = \mathbf{U} = \bar{p} + 3 * \sqrt{(\bar{p}) * (\bar{q}) / n}$$

Let d be the no. of defectives observed in the sample under process control then,d/n < **U**.

If the data satisfy product control then $d \leq c$ for SSP i.e. $d/n \leq c/n$

Case 1: d/n< **U** < c/n ⇒ satisfy process control and product control.

Case 2:d/n<**U**= c/n ⇒ satisfy process control and satisfy product control.

Case 3: d/n<**U** > c/n ⇒satisfy process control and not product control. Hence process and product control occurs only if UCL ≤ MAPD.